

# OSCAR: A visionary, new computer algebra system

William Hart, Sebastian Gutsche  
Reimer Behrends, Thomas Breuer

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*Develop a visionary, next generation, open source computer algebra system, integrating all systems, libraries and packages developed within the TRR.*

**GAP:** computational discrete algebra, group and representation theory, general purpose high level interpreted programming language.

julia

**Singular:** polynomial computations, with emphasis on algebraic geometry, commutative algebra, and singularity theory.

Examples:

- Multigraded equivariant Cox ring of a toric variety over a number field
- Graphs of groups in division algebras
- Matrix groups over polynomial rings over number field

julia

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## Oscar

**polymake:** convex polytopes, polyhedral and stacky fans, simplicial complexes and related objects from combinatorics and geometry.

julia

**ANTIC:** number theoretic software featuring computations in and with number fields and generic finitely presented rings.

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  - ▶ Flint - polynomials and linear algebra over concrete rings
  - ▶ Nemo.jl - Finitely presented rings in Julia
  - ▶ Singular.jl - Julia/Singular integration

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  - ▶ Low-level infrastructure

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  - ▶ Parallelisation
  - ▶ Low-level infrastructure
- ▶ Thomas Breuer - RWTH Aachen
  - ▶ Julia in Gap
  - ▶ Representation theory

- ▶ Hecke: Claus Fieker, Tommy Hofmann, Carlo Sircana

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We are looking for projects that:

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- ▶ Relevant to the TRR

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## Julia libraries:

- ▶ Nemo.jl - generic, finitely presented rings
- ▶ Hecke.jl - number fields, class field theory, algebraic number theory

- ▶ Quadratic sieve integer factorisation

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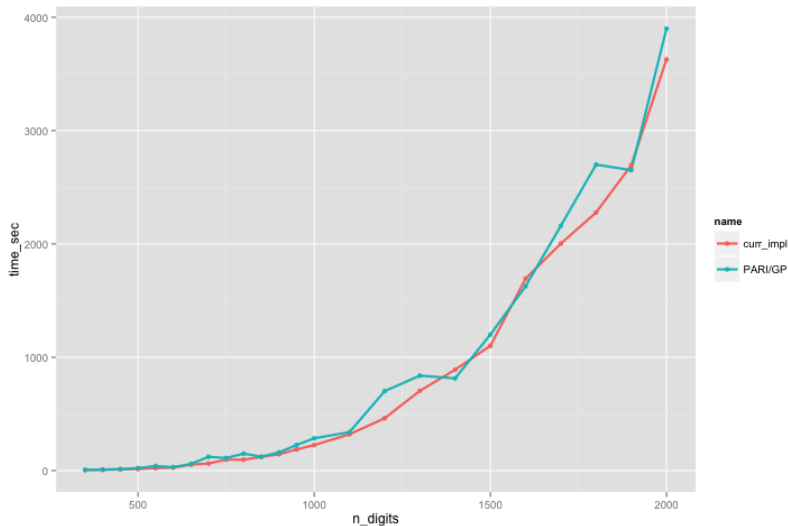
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- ▶ Characteristic and minimal polynomial
- ▶ van Hoeij factorisation for  $\mathbb{Z}[x]$
- ▶ Multivariate polynomial arithmetic  $\mathbb{Z}[x, y, z, \dots]$

Table: Quadratic sieve timings

Digits	Pari/GP	Flint (1 core)	Flint (4 cores)
50	0.43	0.55	0.39
59	3.8	3.0	1.7
68	38	21	14
77	257	140	52
83	2200	1500	540

# APRCL primality test timings



# FFT: Integer and polynomial multiplication

Table: FFT timings

Words	1 core	4 cores	8 cores
110k	0.07s	0.05s	0.05s
360k	0.3s	0.1	0.1s
1.3m	1.1s	0.4s	0.3s
4.6m	4.5s	1.5s	1.0s
26m	28s	9s	6s
120m	140s	48s	33s
500m	800s	240s	150s

# Characteristic and minimal polynomial

Table: Charpoly and minpoly timings

Op	Sage 6.9	Pari 2.7.4	Magma 2.21-4	Giac 1.2.2	Flint
Charpoly	0.2s	0.6s	0.06s	0.06s	0.04s
Minpoly	0.07s	>160 hrs	0.05s	0.06s	0.04s

for  $80 \times 80$  matrix over  $\mathbb{Z}$  with entries in  $[-20, 20]$  and minpoly of degree 40.

# Multivariate multiplication

Table: “Dense” Fateman multiply bench

n	Sage	Singular	Magma	Giac	Piranha	Trip	Flint
5	0.0063s	0.0048s	0.0018s	0.00023s	0.0011s	0.00057s	0.00023s
10	0.51s	0.11s	0.12s	0.0056s	0.029s	0.023s	0.0043s
15	9.1s	1.4s	1.9s	0.11s	0.39s	0.21s	0.045s
20	75s	21s	16s	0.62s	2.9s	2.3s	0.48s
25	474s	156s	98s	2.8s	14s	12s	2.3s
30	1667s	561s	440s	14s	56s	41s	10s

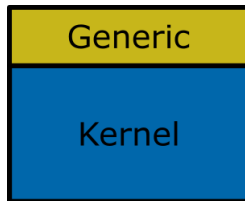
4 variables

# Multivariate multiplication

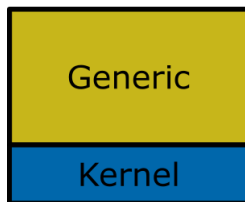
Table: Sparse multiply benchmark

n	Sage	Singular	Magma	Giac	Piranha	Trip	Flint
4	0.0066s	0.0050s	0.0062s	0.0046s	0.0033s	0.0015s	0.0014s
6	0.15s	0.11s	0.080s	0.030s	0.025s	0.016s	0.016s
8	1.6s	0.79s	0.68s	0.28s	0.15s	0.10s	0.10s
10	8s	3.6s	3.0s	1.5s	0.62s	0.40s	0.48s
12	43s	14s	11s	4.8s	2.2s	2.2s	2.0s
14	173s	63s	37s	14s	6.7s	12s	7.2s
16	605s	201s	94s	39s	20s	39s	19s

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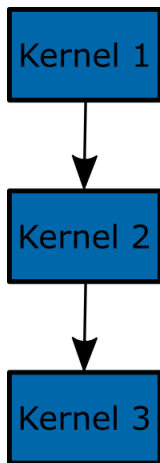
Fast generics



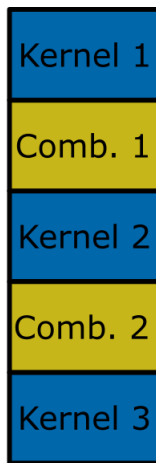
Slow generics



# Efficient generics



Fast data  
transform



Generic  
bottleneck





- ▶ JIT compilation : near C performance.
- ▶ Designed by mathematically minded people.
- ▶ Open Source (MIT License).
- ▶ Actively developed since 2009.
- ▶ Supports Windows, OSX, Linux, BSD.
- ▶ Friendly C/Python-like (imperative) syntax.





Interfaces to C libraries:

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Highlights:

Generic polynomial resultant, charpoly, minpoly over an integrally closed domain, Smith and Hermite normal form, Popov form, fast generic determinant, fast sparse multivariate arithmetic

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- ▶ Data structures for Julia objects and functions in GAP
- ▶ Possibility to add compiled Julia functions as kernel functions to GAP

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- ▶ Nested lists of the above to Arrays

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```
function orbit( self, element, generators, action )
  work_set = [ element ]
  return_set = [ element ]
  generator_length = gap_LengthPlist(generators)
  while length(work_set) != 0
    current_element = pop!(work_set)
    for current_generator_number = 1:generator_length
      current_generator = gap_ListElement(generators,
                                          current_generator_number)
      current_result = gap_CallFunc2Args(action,current_element,
                                         current_generator)

      is_in_set = false
      for i in return_set
        if i == current_result
          is_in_set = true
          break
        end
      end
      if ! is_in_set
        push!( work_set, current_result )
        push!( return_set, current_result )
      end
    end
  end
  return return_set
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gap> orbit_c( 1, S, OnPoints );; time;  
46
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- ▶ providing sufficient amount of integration of GAP data types on the Julia side
- ▶ unifying GAP and Julia memory management



- ▶ Both GAP and Julia use garbage collection for memory management.

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# Coordinating garbage collection for GAP and Julia

- ▶ Both GAP and Julia use garbage collection for memory management.
- ▶ Garbage collection: At intervals, find out which objects aren't in use anymore and throw them away.
- ▶ Problem: GAP and Julia have two distinct, incompatible implementations of garbage collection.
- ▶ Without additional work, objects may be freed prematurely, leading to memory corruption.

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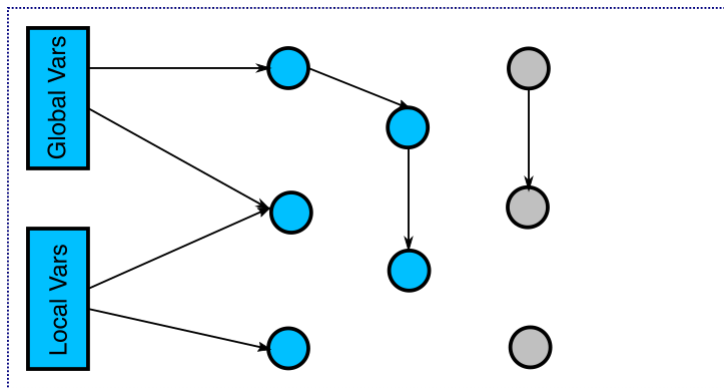
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- ▶ Find every object reachable from a *root*.
- ▶ Dispose of objects that could not be reached.
- ▶ Roots are:
  - ▶ Global variables (static memory).
  - ▶ Local variables and temporary values (stack, registers).



# Example



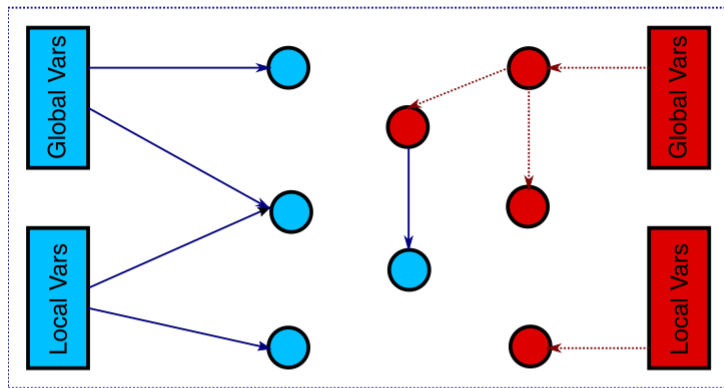
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- ▶ Result: GAP or Julia objects may be freed prematurely.

# Example



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## Pros:

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## Cons:

- ▶ Avoidable inefficiencies (multiset implementation).
- ▶ Unreachable cycles that involve both GAP and Julia objects will not be reclaimed (potential memory leak).

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- ▶ It is possible to use Julia's GC for GAP (with some modifications).
- ▶ GAP supports *almost* everything the Julia GC requires.
- ▶ Exception: root scanning.
  - ▶ Julia's GC determines local variable roots *precisely*.
  - ▶ GAP's GC assumes *conservative* scanning for local variables.



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- ▶ Scan the entire stack and CPU registers word by word.
- ▶ Anything that *may* be a pointer to an object is treated like one.
- ▶ Overly conservative in keeping objects alive.
- ▶ GAP needs conservative scanning, but Julia doesn't support it.

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  1. Can mostly be derived from Julia's data structures
  2. For some cases this needs to be tracked in a separate data structure
- ▶ We have a proof-of-concept implementation.

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- ▶ Requires modified versions of GAP and Julia.



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- ▶ Pursue solutions A and B in parallel.
- ▶ Solution A is minimally invasive and is already used in JuliaInterface.
- ▶ We have a partial prototype for solution B.
- ▶ Next step: Production-ready version of solution B as a minimal patch for Julia/GAP.

From GAP's point of view, Julia can provide

- ▶ new functionality
- ▶ speedup via reimplementing pieces of GAP code in Julia
- ▶ eventually an alternative to parts of GAP?

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Classical recommendation:

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(Is it easy enough for GAP programmers to take this approach?)

# Which parts of GAP are suitable for this approach?

“Low level”:

few calls to GAP functions,  
long nested loops over simple objects

(why not also GAP's C code?)

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- ▶ your suggestions?